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Laminar Fully Developed Flow in Periodically Converging–Diverging Microtubes

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Laminar fully developed flow and pressure drop in linearly varying cross-sectional converging–diverging microtubes have been investigated in this work. These microtubes are formed from a series of converging–diverging modules. An analytical model is developed for frictional flow resistance assuming parabolic axial velocity profile in the diverging and converging sections. The flow resistance is found to be only a function of geometrical parameters. To validate the model, a numerical study is conducted for the Reynolds number ranging from 0.01 to 100, for various taper angles, from 2 to 15 degrees, and for maximum–minimum radius ratios ranging from 0.5 to 1. Comparisons between the model and the numerical results show that the proposed model predicts the axial velocity and the flow resistance accurately. As expected, the flow resistance is found to be effectively independent of the Reynolds number from the numerical results. Parametric study shows that the effect of radius ratio is more significant than the taper angle. It is also observed that for small taper angles, flow resistance can be determined accurately by applying the locally Poiseuille flow approximation.

INTRODUCTION

There are numerous instances of channels that have streamwise-periodic cross sections. It has been experimentally and numerically observed that the entrance lengths of fluid flow and heat transfer for such streamwise-periodic ducts are much shorter than those of plain ducts, and quite often, three to five cycles can make both the flow and heat transfer fully developed [1]. In engineering practice the streamwise length of such ducts is usually much longer than several cycles; therefore, theoretical works for such ducts often focus on the periodically fully developed fluid flow and heat transfer. Rough tubes or channels with ribs on their surfaces are examples of streamwise-periodic ducts that are widely used in the cooling of electronic equipment and gas turbine blades, as well as in high-performance heat exchangers.

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Many researchers have conducted experimental or numerical investigations on the flow and heat transfer in streamwise-periodic wavy channels. Most of these works are based on numerical methods. Sparrow and Prata [1] performed a numerical and experimental investigation for laminar flow and heat transfer in a periodically converging–diverging conical section for the Reynolds number range from 100 to 1000. They showed that the pressure drop for the periodic converging–diverging tube is considerably greater than for the straight tube, while Nusselt number depends on the Prandtl number. For $Pr < 1$, the periodic tube Nu is generally lower than the straight tube, but for $Pr > 1$, Nu is slightly greater than for a straight tube. Wang and Vanka [2] used a numerical scheme to study the flow and heat transfer in periodic sinusoidal passages. Their results revealed that for steady laminar flow, pressure drop increases more significantly than heat transfer. The same result is reported in Niceno and Nobile [3] and Wang and Chen [4] numerical works for the Reynolds number range from 50 to 500. Hydrodynamic and thermal characteristics of a pipe with periodically converging–diverging cross section were investigated by Mahmud et al. [5], using a finite-volume method. A correlation was proposed for calculating the friction factor, in sinusoidal wavy tubes for Reynolds number ranging from 50 to 2,000. Stalio and Piller [6], Bahaidarah [7], and Naphon [8] also studied the flow and heat transfer of periodically varying cross-section channels. An experimental investigation on the laminar flow and mass transfer characteristics in an axisymmetric sinusoidal wavy-walled tube was carried out by Nishimura et al. [9]. They focused on the transitional flow at moderate Reynolds numbers (50 to 1,000). Russ and Beer [10] also studied heat transfer and flow in a pipe with sinusoidal wavy surface. They used both numerical and experimental methods in their work for the Reynolds number range of 400 to 2,000, where the flow regime is turbulent.

For low Reynolds numbers, Re $\sim 0(1)$, some analytical and approximation methods have been carried out in the case of gradually varying cross section. In particular, Burns and Parkes [11] developed a perturbation solution for the flow of viscous fluid through axially symmetric pipes and symmetrical channels with sinusoidal walls. They assumed that the Reynolds number is small enough for the Stokes flow approximation to be made and found stream functions in the form of Fourier series. Manton [12] proposed the same method for arbitrary shapes. Langlois [13] analyzed creeping viscous flow through a circular tube of arbitrary varying cross section. Three approximate methods were developed with no constriction on the variation of the wall. MacDonald [14] and more recently Brod [15] have also studied the flow and heat transfer through tubes of nonuniform cross section.

The low Reynolds number flow regime is the characteristic of flows in microchannels [16]. Microchannels with converging–diverging sections maybe fabricated to influence cross-stream mixing [17–20] or result from fabrication processes such as micromachining or soft lithography [21].

Existing analytical models provide solutions in a complex format, generally in a form of series, and are not amicable to engineering or design. Also, existing model studies did not include direct comparison with numerical or experimental data. In this study, an approximate analytical solution has been developed for velocity profile and pressure drop of laminar, fully developed, periodic flow in a converging–diverging microtube, and results of the model are compared with those of an independent numerical method. Results of this work can be then applied to more complex wall geometries.

PROBLEM STATEMENT

Consider an incompressible, constant property, Newtonian fluid which flows in steady, fully developed, pressure-driven laminar regime in a fixed cross section tube of radius a_0. At the origin of the axial coordinate, $z = 0$, the fluid has reached a fully developed Poiseuille velocity profile, $u(r) = 2u_{m,0}[1 - (\frac{r}{a_0})^2]$, where $u_{m,0}$ is the average velocity. The cross-sectional area for flow varies linearly with the distance z in the direction of flow, but retains axisymmetric about the z-axis. Figure 1 illustrates the geometry and the coordinates for a converging tube; one may similarly envision a diverging tube.

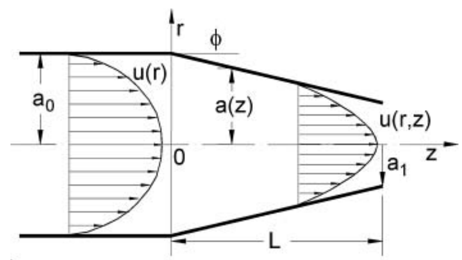

Figure 1 Geometry of slowly varying cross-section microtube.

The governing equations for this two-dimensional (2-D) flow are:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0 \tag{1}$$

$$\rho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{\partial^z u}{\partial z^2}\right\} \tag{2}$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right] + \frac{\partial^2 v}{\partial z^2}\right\} \tag{3}$$

with boundary conditions

$$u(r,z) = 0,\ v(r,z) = 0; \quad r = a(z) \tag{4}$$

$$u(r,0) = 2u_{m,0}\left[1 - \left(\frac{r}{a_0}\right)^2\right]; \quad z = 0$$

$$P(r,0) = P_0$$

In this work we seek an approximate method to solve this problem.

MODEL DEVELOPMENT

The premise of the present model is that the variation of the duct cross section with the distance along the direction of the flow is sufficiently gradual that the axial component of the velocity profile $u(r, z)$ remains parabolic. To satisfy the requirements of the continuity equation, the magnitude of the axial velocity must change, i.e.,

$$u(r,z) = 2u_m(z)\left[1 - \left(\frac{r}{a(z)}\right)^2\right] \tag{5}$$

where $u_m(z)$ is the mean velocity at the axial location z and can be related to the mean velocity $u_{m,0}$ at the origin $z = 0$, and using conservation of mass as

$$u_m(z) = \left[\frac{a_0}{a(z)}\right]^2 u_{m,0} \tag{6}$$

Then the axial velocity profile $u(r, z)$ becomes

$$u(r,z) = 2u_{m,0}\left[\frac{a_0}{a(z)}\right]^2\left[1 - \left(\frac{r}{a\,(z)}\right)^2\right] \tag{7}$$

Substituting Eq. (7) into the continuity equation, Eq. (1), and integrating leads to

$$v(r,z) = 2m\eta u_{m,0}\left[\frac{a_0}{a(z)}\right]^2\left[1-\left(\frac{r}{a\,(z)}\right)^2\right] \tag{8}$$

where $m = \frac{da(z)}{dz}$ is the wall slope and $\eta = \frac{r}{a(z)}$.

PRESSURE DROP AND FLOW RESISTANCE

Comparing Eqs. (7) and (8) reveals that $\frac{v}{u} = m\eta$; thus, one can conclude that if m is small enough, v will be small and the pressure gradient in the r direction can be neglected with respect to pressure gradient in the z direction.

Knowing both velocities and neglecting $\frac{\partial P}{\partial r}$, pressure drop in a converging–diverging module can be obtained by integrating Eq. (2). The final result after simplification is

$$\Delta P = \frac{16\mu u_{m,0}L}{a_0^2}\left[\frac{\varepsilon^2+\varepsilon+1}{3\varepsilon^2}+\frac{m^2(1+\varepsilon)}{2\varepsilon^5}\right] \tag{9}$$

where ΔP is the difference of average pressure at the module inlet and outlet, a_0 and a_1 are the maximum and minimum radiuses of the tube, respectively, $m = \tan\phi$ is the slope of the tube wall, and $\varepsilon = \frac{a_1}{a_0}$ is the minimum–maximum radius ratio.

Defining flow resistance with an electrical network analogy in mind [22],

$$R_f = \frac{\Delta P}{Q} \tag{10}$$

where $Q = \pi a_0^2 u_{m,0}$, the flow resistance of a converging–diverging module becoms

$$R_f = \frac{16\mu L}{\pi a_0^4}\left[\frac{\varepsilon^2+\varepsilon+1}{3\varepsilon^2}+\frac{m^2(1+\varepsilon)}{2\varepsilon^5}\right] \tag{11}$$

At the limit when $m = 0$, Eq. (11) recovers the flow resistance of a fixed-cross-section tube of radius a_0, i.e.

$$R_{f,0} = \frac{16\mu L}{\pi a_0^4} \tag{12}$$

In dimensionless form,

$$R_f^* = \left[\frac{\varepsilon^2+\varepsilon+1}{3\varepsilon^2}+\frac{m^2(1+\varepsilon)}{2\varepsilon^5}\right] \tag{13}$$

The concept of flow resistance, Eq. (10), can be applied to complex geometries by constructing resistance networks to analyze the pressure drop.

For small taper angles ($\phi \le 10^\circ$), the term containing m^2 becomes small, and thus Eq. (13) reduces to

$$R_f^* = \frac{\varepsilon^2+\varepsilon+1}{3\varepsilon^2} \tag{14}$$

The maximum difference between the dimensionless flow resistance, R_f^*, obtained from Eq. (13) and that from Eq. (14) is 6% for $\phi = 1^\circ$. Equation (14) can also be derived from the locally Poiseuille approximation. With this approximation, the frictional resistance of an infinitesimal element in a gradually varying cross-section microtube is assumed to be equal to the flow resistance of that element with a straight wall. Equation (14) is used for comparisons with numerical data.

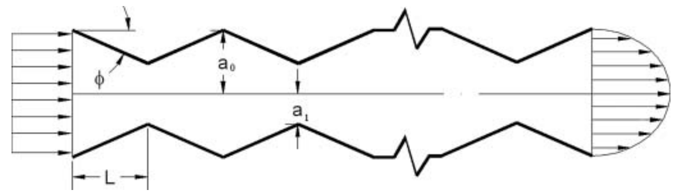

Figure 2 Schematic of the periodic converging–diverging microtube.

NUMERICAL ANALYSIS

To validate the present analytical model, 15 modules of converging–diverging tubes in a series were created in a finite-element-based commercial code, COMSOL 3.2 (www.comsol.com). Figure 2 shows the schematic of the modules considered in the numerical study. Two geometrical parameters, taper angle, ϕ, and minimum–maximum radius ratio, $\varepsilon = \frac{a_1}{a_0}$, were varied from 0 to 15° and 0.5 to 1, respectively. The working fluid was considered to be Newtonian with constant fluid properties. A Reynolds number range from 0.01 to 100 was considered. Despite the model is developed based on the low Reynolds numbers, higher Reynolds numbers (Re $\sim$ 100) were also investigated to evaluate the limitations of the model with respect to the flow condition. A structured, mapped mesh was used to discretize the numerical domain. Equations (1)–(3) were solved as the governing equations for the flow for steady-state condition. A uniform velocity boundary condition was applied to the flow inlet. Since the flow reaches streamwise fully developed condition in a small distance from the inlet, the same boundary conditions as Eq. (4) can be found at each module inlet. A fully developed boundary condition was assumed for the outlet, $\frac{\partial}{\partial z} = 0$. A grid refinement study was conducted to ensure accuracy of the numerical results. Calculations were performed with grids of 3×6, 6×12, 12×24, and 24×48 for each module for various Reynolds numbers and geometrical configurations. The value of dimensionless flow resistance, R_f^*, was monitored since the velocity profile in any cross section remained almost unchanged with the mesh refinement. Figure 3 shows the effect of mesh resolution on R_f^* for $\phi = 10^\circ$, $\varepsilon = \frac{a_1}{a_0} = 0.95$, and Re $= 10$. As can be seen, the value of R_f^* changes slower when the mesh resolution increases. The fourth mesh, i.e., 24×48, was considered in this study for all calculations to optimize computation cost and the solution accuracy.

The effect of the streamwise length on the flow has been shown in Figures 4 and 5. Dimensionless velocity profile, $u^* = \frac{u}{u_{\max(z)}}$, is plotted at $\beta = \frac{a_0}{a(z)} = 1.025$ for the second to fifth

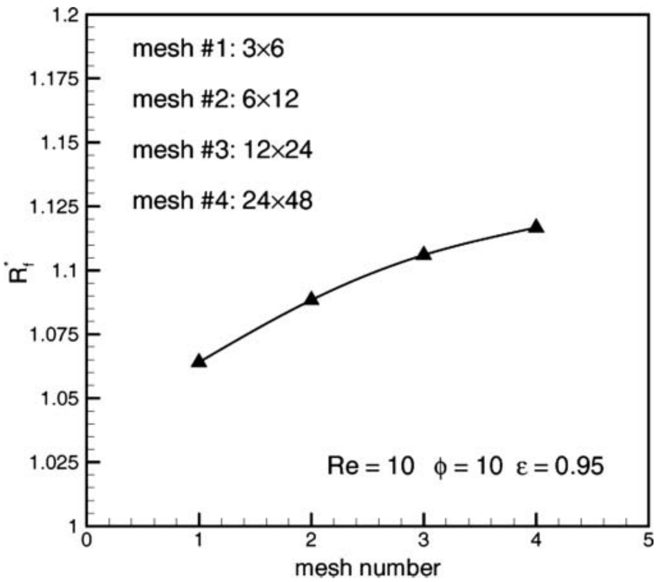

Figure 3 Mesh independency analysis.

modules as well as the dimensionless flow resistance, R_f^* for the second to seventh modules for the typical values of $\phi = 10^\circ$, $\varepsilon = \frac{a_1}{a_0} = 0.95$, and Re = 10. Both velocity profile and dimensionless flow resistance do not change after the forth module, which indicates that the flow after the fourth module is fully developed. The same behavior was observed for the geometrical parameters and Reynolds numbers considered in this work. Values of the modules in the fully developed region were used in this work.

Good agreement between the numerical and analytical model can be seen in Figure 6, where the dimensionless frictional flow resistance, R_f^*, is plotted over a wide range of the Reynolds number, $Re = \frac{2\rho u_{m,0} a_0}{\mu}$. The upper and lower dashed lines represent the bounds of nondimensional flow resistance for the investigated microtube. $R_{f,0}^*$ is the flow resistance of a uniform

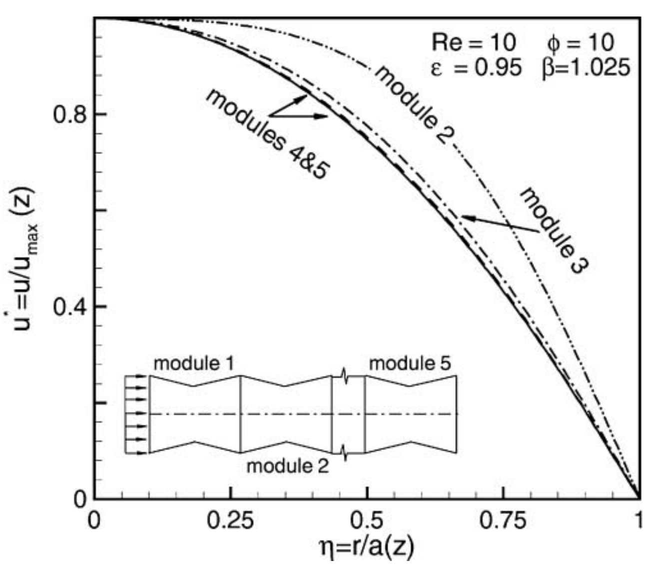

Figure 4 Effect of the streamwise length.

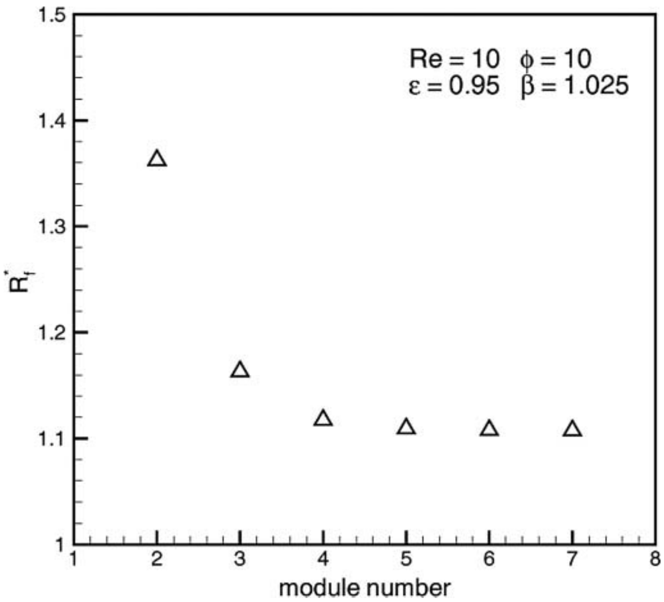

Figure 5 Effect of module number on the dimensionless flow resistance.

cross-sectional tube with the radius of a_0, and as expected its value is unity. $R_{f,1}^*$ stands for the flow resistance of a tube with the radius of a_1. Since the average velocity is higher for the tube of radius a_1, the value of $R_{f,1}^*$ is higher than the value of $R_{f,0}^*$. Both numerical and analytical results show the flow resistance to be effectively independent of Reynolds number, in keeping with low Reynolds number theory. For low Reynolds numbers, in the absence of instabilities, flow resistance is independent of the Reynolds number.

Table 1 lists the comparison between the present model, Eq. (14), and the numerical results over the wide range of minimum–maximum radius ratio, $0.5 \leq \varepsilon \leq 1$, three typical Reynolds numbers of Re = 1, 10, and 100, and taper angles of $\phi = 2.7^\circ$ and 15°. The model is originally developed for small wall taper angles, $\phi \leq 10$, and low Reynolds numbers, Re $\sim 0(1)$; however,

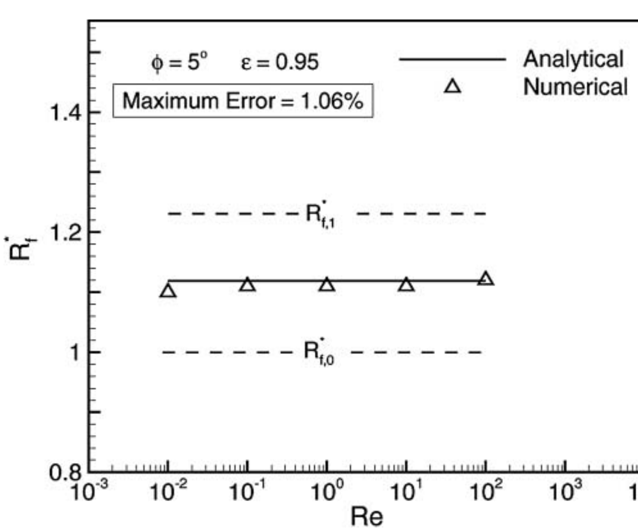

Figure 6 Variation of R_f^* with the Reynolds number, $\phi = 10$, and $\varepsilon = 0.95$.

Table 1 Comparison of the proposed model and the numerical results

		Re = 1		Re = 10		Re = 100	
ε	Model	Numerical	Error (%)	Numerical	Error (%)	Numerical	Error (%)
ϕ = 2							
0.5	4.67	4.59	–1.7	4.59	–1.7	4.96	+6.2
0.6	3.02	2.97	–1.7	2.98	–1.7	3.14	+3.7
0.7	2.13	2.09	–1.7	2.09	–1.7	2.17	+1.9
0.8	1.59	1.56	–1.8	1.56	–1.8	1.59	+0.0
0.9	1.24	1.21	–2.2	1.22	–1.6	1.23	–0.8
1	1	1	0.0	1	0.0	1	0.0
ϕ = 7							
0.5	4.67	4.67	0.0	4.72	+1.2	6.00	+28.6
0.6	3.02	3.02	0.0	3.05	+0.9	3.55	+17.2
0.7	2.13	2.13	–0.2	2.14	+0.7	2.33	+9.7
0.8	1.59	1.59	–0.6	1.60	+0.7	1.66	+4.5
0.9	1.24	1.24	0.0	1.24	0.0	1.25	+0.6
1	1	1	1.0	1	0.0	1	0.0
ϕ = 15							
0.5	4.67	5.01	+7.3	5.25	+12.4	7.33	+57.0
0.6	3.02	3.24	+7.3	3.33	+10.0	4.11	+36.0
0.7	2.13	2.26	+6.2	2.32	+8.9	2.57	+20.7
0.8	1.59	1.67	+5.4	1.70	+6.7	1.76	+10.7
0.9	1.24	1.28	+3.4	1.29	+3.7	1.32	+6.8
1	1	1	0.0	1	0.0	1	0.0

$Re = \frac{2\rho u_{m,0} a_0}{\mu}$ $Error\% = \frac{R^*_{f,\,model} - R^*_{f,\,numerical}}{R^*_{f,\,model}}$

as can be seen in Table 1, the proposed model can be used for wall taper angles up to 15°, when Re < 10, with acceptable accuracy. Note that the model shows good agreement with the numerical data for higher Reynolds numbers, up to 100, when $\varepsilon > 0.8$. Instabilities in the laminar flow due to high Reynolds numbers and/or large variations in the microchannel cross section result in the deviations of the analytical model from the numerical data.

PARAMETRIC STUDIES

Effects of two geometrical parameters—minimum–maximum radius ratio, ε, and taper angle, ϕ—are investigated and shown in Figures 7 and 8. Input parameters of two typical converging–diverging microtube modules are shown in Table 2. In the first case, the effect of $\varepsilon = \frac{a_1}{a_0}$ on the flow resistance was studied when taper angle, $\phi = 7$, was kept constant. As shown in Figure 7, both numerical and analytical results indicate that the frictional flow resistance, R_f, decreases by increasing of the minimum–maximum radius ratio, ε. For a constant taper angle, increase of $\varepsilon = \frac{a_1}{a_0}$ increases the module length as well as the average fluid velocity. Hence, higher flow resistance can be observed in Figure 7 for smaller values of ε. For better physical interpretation, flow resistances of two straight microtubes

Table 2 Input parameters for two typical microtubes

Parameter	Value
a_0	500 μm
Re	10
Case 1	
$\phi = 7$	
$0.5 < \varepsilon < 1$	
Case 2	
$\varepsilon = 0.8$	
$2 \le \phi \le 15$	

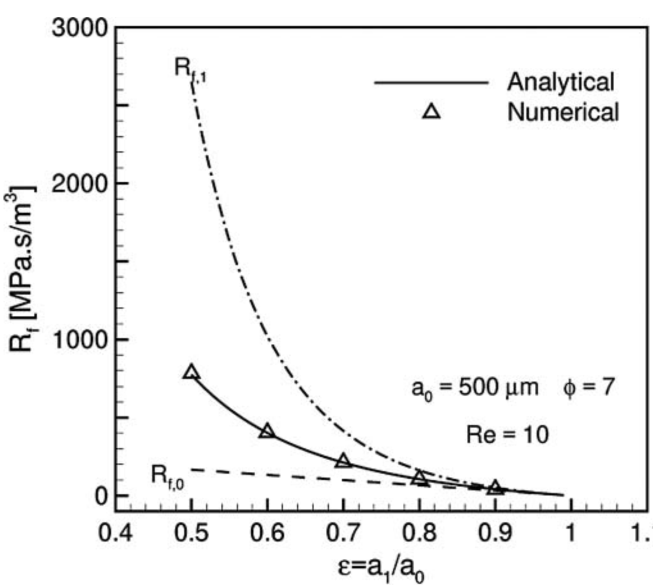

Figure 7 Effect of ε on the flow resistance, $\phi = 7$, and Re = 10.

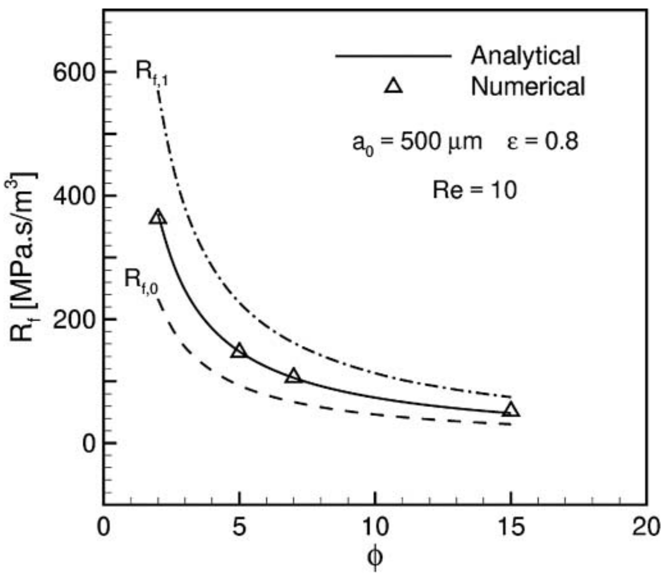

Figure 8 Effect of ϕ on the flow resistance, $\varepsilon = 0.8$, and Re = 10.

with the maximum and minimum module radiuses are plotted in Figure 7. Since the total length of the module increases inversely with ε, a slight increase in $R_{f,0}$ can be observed. On the other hand, the flow resistance of the microtube with the minimum radius of the module $R_{f,1}$ increases sharply when ε becomes smaller. Keeping in mind that the flow resistance is inversely related to the fourth power of the radius, Eq. (12), and a_1 changes with ε, sharp variation of $R_{f,1}$ can be observed in Figure 7.

Variation of the flow resistance with respect to the taper angle when the minimum–maximum radius ratio, $\varepsilon = \frac{a_1}{a_0}$, was kept constant is plotted in Figure 8. Since a_1 remains constant in this case, the only parameter that has an effect on the flow resistance is the variation of the module length with respect to the taper

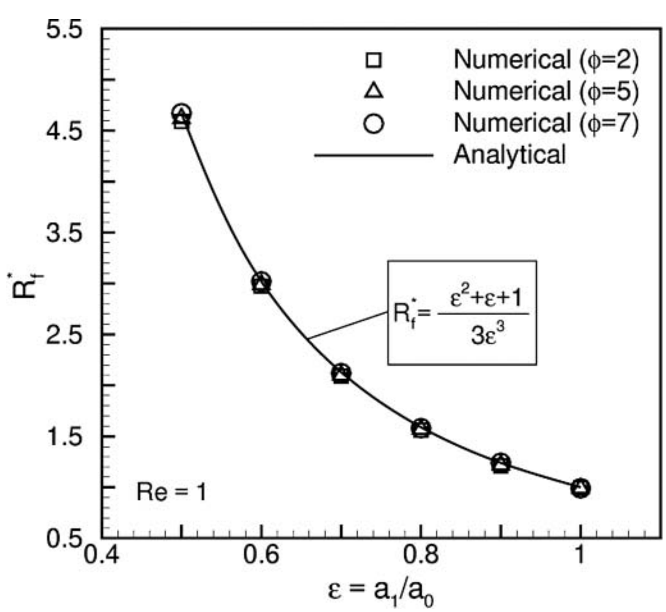

Figure 9 Effect of ϕ and ε on R_f^*, Re = 1.

angle. Both $R_{f,0}$ and $R_{f,1}$ increase inversely with the taper angle ϕ in a similar manner.

The effect of the module length can be eliminated by nondimensionalizing the module flow resistance with respect to the flow resistance of a straight microtube. Dimensionless flow resistance with the definition of Eq. (12) was used in Figure 9. As can be seen, the taper angle ϕ effect is negligible while the controlling parameter is the minimum–maximum radius ratio, ε.

SUMMARY AND CONCLUSIONS

Laminar fully developed flow and pressure drop in gradually varying cross-sectional converging–diverging microtubes have been investigated in this work. A compact analytical model has been developed by assuming that the axial velocity profile remains parabolic in the diverging and converging sections. To validate the model, a numerical study has been performed. For the range of Reynolds number and geometrical parameters considered in this work, numerical observations show that the parabolic assumption of the axial velocity is valid. The following results are also found through analysis:

- For small taper angles ($\phi \leq 10$), effect of the taper angle on the dimensionless flow resistance, R_f^* can be neglected with less than 6% error and the local Poiseuille approximation can be used to predict the flow resistance.
- It has been observed through the numerical analysis that the flow becomes fully developed after less than five modules of length.
- Comparing the present analytical model with the numerical data shows good accuracy of the model to predict the flow resistance for Re < 10, $\phi \leq 10$, and $0.5 \leq \varepsilon \leq 1$. See Table 1 for more details.
- The effect of minimum–maximum radius ratio, ε, is found to be more significant than taper angle, ϕ on the frictional flow resistance.

As an extension of this work, an experimental investigation to validate the present model and numerical analysis is in progress.

NOMENCLATURE

$a(z)$	=	radius of tube, m
a_0	=	maximum radius of tube, m
a_1	=	minimum radius of tube, m
L	=	half of module length, m
m	=	slope of tube wall, [—]
Q	=	volumetric flow rate, m^3/s
r, z	=	cylindrical coordinate, m
Re	=	Reynolds number, $\frac{2\rho u_{m,0} a_0}{\mu}$
R_f	=	frictional resistance, $pa\frac{s}{m^3}$
R_f^*	=	normalized flow resistance, $\frac{R_f}{R_{f,0}}$
u_m	=	mean fluid axial velocity, m/s
u, v	=	velocity in z and r directions, m/s

Greek Symbols

β	=	$\frac{a_0}{a(z)}$
η	=	$\frac{r}{a(z)}$
ε	=	$\frac{a_1}{a_0}$
ρ	=	fluid density, kg/m^3
μ	=	fluid viscosity, kg/m-s
ϕ	=	angle of tube wall, [—]
ΔP	=	pressure drop, Pa

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